1948

Analysis of simultaneous faults by the method of symmetrical components

Robert Marshall Montgomery

ANALYSIS OF SIMULTANEOUS FAULTS
BY THE METHOD OF SYMMETRICAL COMPONENTS

BY

ROBERT M. MONTGOMERY

A

THESIS

submitted to the faculty of the

SCHOOL OF MINES AND METALLURGY OF THE UNIVERSITY OF MISSOURI

in partial fulfillment of the work required for the

Degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

Rolla, Missouri

1948

Approved by F. H. Frame

Professor of Electrical Engineering

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

LIST OF TABLES

INTRODUCTION

The object of this thesis is to present a method for the analytic determination of fault currents and voltages of a symmetrical three-phase system that is subjected to three simultaneous faults. The fault analysis is to be effected by the method of symmetrical components.

The method of symmetrical components, which includes the replacement of any system of three vectors by three sets of balanced vectors, is now used extensively in the analysis of fault conditions on three-phase systems for faults occurring at one, two or more points. For one and two fault points the method has been outlined and developed in various texts and articles. The less common and more obscure case of three simultaneously occurring faults, though essentially an extension of the method used for two faults, still remains unpublished so far as can be ascertained.

Simultaneous faults may be produced on separate circuits, physically close together, by the same lightning disturbance; or a single line to ground fault on one phase may raise the voltage to such an extent as to cause flashovers, or line to line faults, at a second or third point.

Though relatively infrequent, simultaneous faults are important because relay systems, operating satisfactorily for single faults, may fail to isolate simultaneous faults. Therefore, it seems worthwhile to have in convenient form a method for calculating short-circuit currents and voltages caused by faults occurring at the same time at three separate and distinct points of a system.

It is the purpose of this paper to outline this method in detail, developing the equations and constants necessary for a ready calculable solution.

REVIEW OF LITERATURE

The entire history of the development of the Method of Symmetrical Components has been connected with polyphase rotating machines and with the efforts to discover a method for determining their characteristics when operating under unbalanced conditions.

The Method of Symmetrical Components had its origin in the mathematical analysis of induction machines operating under unbalanced conditions. Among the early analysers of single-phase motors were Ferraris(1) and Lamme(2) who recognized that the field set up in the single-phase motor could be resolved into two revolving flux fields rotating in opposite directions.

Somewhat later, L. G. Stokvis(3) published a paper indicating the possibility of resolving unbalanced currents from three-phase machines into two sets of components that are now known as positive- and negative-sequence components. These effects were analyzed in terms of the positively and negatively rotating fields in the machine associated with the positive- and negative-sequence components. In 1915 Stokvis published a more detailed analysis(4) of resolution into positive- and negative-se-

---

(1) Ferraris, "Rotazioni elettrodynamiche", Turin Acad., March, 1888.
(2) Lamme, B. B., Jour, A.I.E.E., March 1921.
(3) L. G. Stokvis, R. Oldenbourg, Munich, 1922.
(4) L. G. Stokvis, Compt. Rend., Vol 159, pp.46-49.

quence currents, this being done in connection with the determination of generator voltage regulation in terms of phase currents. Stokvis failed to recognize that a new kind of component (zero-sequence component) was needed, and thus lacked the essential element required to set up components which did not react upon each other in symmetrical parts of the system.

Dr. C. L. Fortescue[5], approaching the problem from a different point of view, introduced the concept of zero-sequence currents and voltages, and thus provided a method for the solution of all kinds of balanced and unbalanced poly-phase systems. The general concept, developed in studying problems of unbalanced circuits and in analysing the characteristics of poly-phase motors with unbalanced voltages, was presented by Fortescue in 1918 before the A. I. E. E. Fortescue proposed to divide the three armature currents, in case their sum was zero (neutral isolated) into positive- and negative-sequence systems of currents as Stokvis had done previously. However, when their sum was not zero, Fortescue proposed to divide the sum of the currents (neutral current) equally among the phases, instead of assuming, as Stokvis had done, that it was confined to one phase alone. Thus, the zero-sequence component of armature currents is the same in mag-

(5) C. L. Fortescue, A.I.E.E. Trans., Vol.34, Part II, 1918, pp. 1027-1140.

nitude and in time-phase in each of the three phases and is equal to one-third of the neutral current. Since the time phase angle in the three phases is zero, thus follows the name zero-sequence component.

Though the discussions in this paper shall be limited to three phases, The Method of Symmetrical Components is perfectly general and can be applied to systems of any number of phases, with similiar results. In his classic paper[6] Fortescue proved that "a system of n vectors or quantities may be resolved when n is prime into n different symmetrical groups or systems, one which consists of n equal vectors and the remaining (n - 1) systems consist of n equi-spaced vectors which with the first mentioned groups of equal vectors forms an equal number of symmetrical n-phase systems...." Thus, any unbalanced three-phase system of vectors may be replaced by (1) a balanced system of three-phase vectors (positive-sequence system) having the same phase sequence as the original unbalanced system of vectors, (2) a balanced system of three-phase vectors (negative-sequence system) having a phase sequence which is opposite to that of the original unbalanced system of vectors, and (3) a system of three single-phase vectors (zero-sequence system) which are equal in magnitude and phase.

---

(6) C. L. Fortescue, A.I.E.E. Trans., Vol.34, Part II, 1918, pp. 1027-1140.

Also Fortescue demonstrated that in those parts of a system which are symmetrical, the currents and voltages of one sequence have no influence upon those of another sequence.

Since 1918 The Method of Symmetrical Components has been used in the solution of many problems, the most important of these being current and voltage analysis under unbalanced fault conditions. The concepts of sequence networks and equivalent circuits have greatly facilitated these studies.

R. D. Evans in 1925 was the first to describe the application of short-circuited systems making use of the direct-current and alternating current calculating boards. The direct-current calculating board[7] may be used to reduce the time required for solution when resistance and capacitance are neglected and the system is operating at no load.

In 1926 R. D. Evans and C. F. Wagner applied the method to system-stability problems. Still further work concerning short-circuit conditions on synchronous machines has been done by Bekku, Wagner and Dovjikov, Park and Robertson, Doherty and Nickle, and Kilgore and Wright. Much study has also been given to the zero-sequence impedance of transmission lines and cables which is of parti-

(7) W. W. Lewis, Gen.Elec.Rev., Aug., 1920, pp.669-671.

cular importance in connection with the problem of inductive interference. Many of the inductive coordination studies[8] which have been made have employed the theory of symmetrical components.

---

(8) "Engineering Reports of the Joint Subcommittee on Development and Research", National Electric Light Association and Bell Telephone System, Vol. I, Engineering Report 4, pp. 7-47.

UNSYMMETRICAL FAULTS ON THREE-PHASE SYSTEMS

The Method of Symmetrical Components as applied to single and double faults will be discussed first, after which the more obscure case of three simultaneously occurring faults will be considered. In all cases the discussions will be limited so as to apply only to balanced, or symmetrical, three-phase systems, or to previously balanced systems which have been rendered unbalanced by the occurence of one or more unsymmetrical faults. A balanced three-phase system is one in which the phase currents and voltages at every point in the system are balanced, and are of the same frequency and phase order as the generated voltages. Four types of short-circuits will be considered, these being:

1. Three-phase fault.
2. Line-to-ground fault.
3. Double line-to-ground fault.
4. Line-to-line fault.

The Method of Symmetrical Components embodies the replacement of a system of three vectors (voltages or currents), regardless of their degree of unbalance, by three sets of balanced vectors. These three sets of balanced vectors, as previously stated, are known as the positive-sequence, negative-sequence, and zero-sequence systems of vectors. For example, any three vector voltages, V_a, V_b, and V_c may be expressed in terms of their symmetrical components by the following equations:

$$V_a = V_{x1} + V_{x2} + V_{xo} \tag{1}$$

$$V_b = V_{y1} + V_{y2} + V_{yo} \tag{2}$$

$$V_c = V_{z1} + V_{z2} + V_{zo} \tag{3}$$

The components may be represented vectorially as shown in Fig. 1. The counterclockwise direction of rotation is

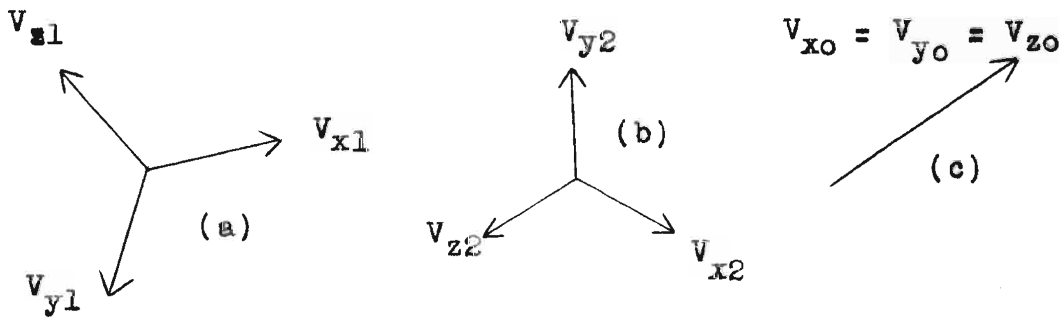


Fig. 1. (a) Positive-, (b) Negative-, and (c) zero-sequence systems of vectors.

arbitrarily taken as positive, so the order of arrival of the positive-sequence component vectors at the reference axis is V_{x1}, V_{y1}, V_{z1}. Since a definite phase relationship exists between the components of the three sequences, equations (1), (2), and (3) may be rewritten in terms of V_{x1}, V_{x2}, and V_{xo} only, where a is the vector operator $1 \underline{/ 120^\circ}$.

$$V_a = V_{x1} + V_{x2} + V_{xo} \tag{4}$$

$$V_b = a^2V_{x1} + aV_{x2} + V_{xo} \tag{5}$$

$$V_c = aV_{x1} + a^2V_{x2} + V_{xo} \tag{6}$$

Simultaneous solution of equations (4), (5), and (6) yields the following results for the symmetrical components of V_a.

$$V_{xo} = 1/3\ (V_a + V_b + V_c) \qquad (7)$$

$$V_{x1} = 1/3\ (V_a + aV_b + a^2V_c) \qquad (8)$$

$$V_{x2} = 1/3\ (V_a + a^2V_b + aV_c) \qquad (9)$$

By expressing the three original vectors in terms of three new vectors, calculations are often greatly simplified and a better understanding of the problem obtained. Thus, since voltages and currents may be replaced by component voltages and currents, it is necessary that the impedances associated with these component voltages and currents be readily determinable, if any simplified solution is to be obtained. It is, indeed, in this fact that the advantage of the Method of Symmetrical Components lies; for the impedances met by the various sequence currents may be determined by calculation or test, whereas this is not always the case with phase impedances. When phase impedances may readily be determined, introducing components may only serve to complicate the solution unless the problem is one involving a complex network.

It should be realized that any method of calculating voltages and currents by adding currents arising from component voltages and voltages existing because of component currents depends for its validity upon the Principle of Superposition. Since the Method of Symmetrical Components incorporates these features, it must, therefore, be justified by the Principle of Superposition. If Superposition is to be rigorously applied, all circuit parameters (inductance, resistance, etc.) must be independent of the current, voltage, and frequency associated with them.

Frequent use will be made of the one-line impedance diagram, by which balanced three-phase systems are represented by equivalent single-phase systems. More explicitly, the one-line diagram represents only one phase of the three-phase system, all currents representing line currents, all voltages line to neutral voltages, and all impedances the phase impedances. The return path for all currents is a conductor (zero-potential bus) of zero impedance at zero potential.

Fortescue proved, for both static and rotating machinery, that in symmetrical systems different sequence components do not react upon each other; thus, positive-sequence voltages produce only positive-sequence currents, negative-sequence voltages produce only negative-sequence currents, and zero-sequence voltages produce only zero-sequence currents. Therefore, each of the three sequence systems may be considered separately, with every component of voltage and current in any sequence being calculated from a consideration of that sequence only.

The positive- and negative-sequence one-line impedance diagrams of a network are, in general, of the same form as the one-line diagram of the network. However, since three-phase alternators are designed to generate balanced voltages, all generated voltages will be positive-sequence voltages. This may be shown as follows: Let the generated voltages of a three-phase alternator be E_a, E_b, and E_c. Then, remembering that the sum of three balanced voltages is zero, substitution in equations (7), (8), and (9) gives:

$$E_{xo} = E_{x2} = 0 \qquad\qquad E_{x1} = E_a$$

Therefore, since all generated voltages are positive-sequence voltages and since positive-sequence voltages produce positive-sequence currents only, the negative- and zero-sequence networks will contain no emf's and will conduct no current so long as the system is balanced. However, an unsymmetrical fault applied to the system may cause a negative- and zero-sequence voltage to appear at the fault point with resulting negative- and zero-sequence currents.

Figure 2 represents part of a symmetrical three-phase system. At point F three fault currents, I_a, I_b, and I_c are shown flowing into hypothetical stub connections of

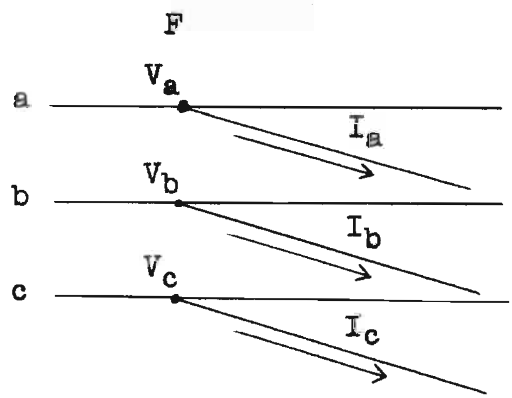


Fig. 2. Fault currents at point F on a three-phase system.

zero impedance. V_a, V_b, and V_c designate the voltages to ground on phases a, b, and c, respectively. The positive-, negative-, and zero-sequence components of I_a are I_{x1}, I_{x2}, and I_{xo}, respectively, and the corresponding components of line-to-ground voltage V_a are V_{x1}, V_{x2}, and V_{xo}.

Before the occurence of a fault, there are no fault currents, and all voltages are balanced, positive-sequence voltages. The effect of a fault on the positive-sequence network is to change the positive-sequence voltage from some prefault value V_f to V_{x1}, and the positive-sequence fault current from zero to I_{x1}. Since no currents nor voltages are present in the negative- and zero-sequence networks before the fault, the effect of the fault on these networks is to change negative- and zero-sequence voltages and currents from zero to V_{x2}, zero to V_{xo}, zero to I_{x2}, and zero to I_{xo}, respectively. If fault currents and voltages are to be determined, six independent equations are necessary to solve for the six, unknown quantities, I_{x1}, I_{x2}, I_{xo}, V_{x1}, V_{x2}, and V_{xo}.

If positive direction of current flow be taken as into the fault, or out of the network, the positive-sequence current arising from the change in voltage $(V_{x1}-V_f)$ is

$$I_{x1} = \frac{-(V_{x1} - V_f)}{Z_1} \qquad (10)$$

where Z_1 is the positive-sequence impedance of the network as viewed from the fault point. Solving equation (10) for V_{x1},

$$V_{x1} = V_f - I_{x1} Z_1 \, . \qquad (11)$$

By the Principle of Superposition the total positive-sequence current may be determined by adding the positive-sequence current existing before the fault to the positive-sequence current resulting from the voltage $(V_{x1}-V_f)$ applied at the fault point.

The positive-sequence voltage at the fault, V_{x1}, may be obtained in a slightly different manner. Since the change in voltage is $(V_{x1} - V_f)$, the positive-sequence fault current that will flow may be determined by applying this change in voltage between the fault point and the zero-potential bus. This voltage causes current to flow from the fault point into the network. Since I_{x1} is defined as the positive-sequence current flowing into the fault, it will produce a voltage rise, $-I_{x1}Z_1$, in flowing from the zero-potential bus, through the network and into the fault. The resulting positive-sequence voltage at the fault is equal to the sum of $-I_{x1}Z_1$ and V_f thus giving the results of equation (11).

The negative-sequence current I_{x2} arising from the change in voltage V_{x2} may be determined by applying this change in voltage between the fault point and the zero-potential bus. Since positive direction of current flow is out of the network and into the fault, the rise in voltage from the zero-potential bus to the fault point is $-I_{x2}Z_2$, where Z_2 is the negative-sequence impedance of the network as viewed from the fault. In equational form

$$V_{x2} = - I_{x2} Z_2 \text{ .} \tag{12}$$

By similiar reasoning the zero-sequence voltage at the fault point is

$$V_{xo} = - I_{xo} Z_o \text{ .} \tag{13}$$

Equations (11), (12), and (13) are three of the six independent equations needed for the solution of the six component quantities. The other three equations must be obtained from a consideration of the conditions at the fault. This will be illustrated for the case of a sin-

$V_a = 0$

$I_b = 0$

$I_c = 0$

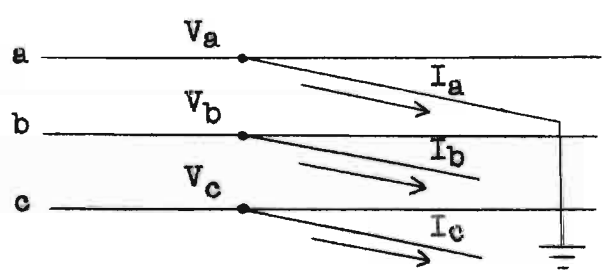


Fig. 3. Single line-to ground fault.

gle line-to-ground fault occurring on phase a, as shown in Figure 3. The conditions existing at the fault are also given in Figure 3.

Substituting $I_b = I_c = 0$ in equations (7), (8), and (9) and replacing voltages by currents,

$$I_{xo} = 1/3(I_a + 0 + 0) = 1/3\ I_a$$

$$I_{x1} = 1/3(I_a + 0 + 0) = 1/3\ I_a$$

$$I_{x2} = 1/3(I_a + 0 + 0) = 1/3\ I_a$$

Therefore,

$$I_{xo} = I_{x1} \qquad (14)$$

$$I_{x2} = I_{x1} \qquad (15)$$

Substituting $V_a = 0$ in equation (4),

$$V_{x1} = -\ (V_{xo} + V_{x2}) \qquad (16)$$

Equations (14), (15), and (16) together with equations (11), (12), and (13) afford the necessary relationships to determine all unknown quantities.

Table I expresses the relations between the symmetrical components of V_a and I_a for four short-circuit type faults. For any short-circuit type fault three equations from Table I together with equations (11), (12), and (13) provide the six independent equations necessary for a solution of the six unknown components.

TABLE I[9]

Fault Equations Expressing Relations Between The Components of I_a and V_a

Phase a is reference phase. I_a is reference phase and V_a is voltage to ground at the fault.

Case A. Line-to-Ground Fault

(a) Phase a		
$I_{xo} = I_{x1}$	$I_{x2} = I_{x1}$	$V_{x1} = -(V_{xo} \quad V_{x2})$
(b) Phase b		
$I_{xo} = a^2 I_{x1}$	$I_{x2} = aI_{x1}$	$V_{x1} = -(aV_{xo} \quad a^2V_{x2})$
(c) Phase c		
$I_{xo} = aI_{x1}$	$I_{x2} = a^2 I_{x1}$	$V_{x1} = -(a^2V_{xo} \quad aV_{x2})$

Case B. Line-to-Line Fault

(a) Phases b and c		
$I_{xo} = 0$	$I_{x2} = -I_{x1}$	$V_{x2} = V_{x1}$
(b) Phases a and c		
$I_{xo} = 0$	$I_{x2} = -aI_{x1}$	$V_{x2} = aV_{x1}$
(c) Phases a and b		
$I_{xo} = 0$	$I_{x2} = -a^2 I_{x1}$	$V_{x2} = a^2 V_{x1}$

Case C. Double Line-to-Ground Fault

(a) Phases b and c		
$I_{x1} = -(I_{xo} \quad I_{x2})$	$V_{xo} = V_{x1}$	$V_{x2} = V_{x1}$
(b) Phases a and c		
$I_{x1} = -(aI_{xo} \quad a^2 I_{x2})$	$V_{xo} = a^2 V_{x1}$	$V_{x2} = aV_{x1}$
(c) Phases a and b		
$I_{x1} = -(a^2 I_{xo} \quad aI_{x2})$	$V_{xo} = aV_{x1}$	$V_{x2} = a^2 V_{x1}$

Case D. Three-phase Fault

(a) Phases a, b, and c		
$I_{xo} = 0$	$V_{x1} = 0$	$V_{x2} = 0$
(b) Phases a, b, c, and ground		
$V_{x1} = 0$	$V_{x2} = 0$	$V_{xo} = 0$

---

(9) Clarke, Edith, Circuit Analysis of A-C Power Systems. Volume I, p 198.

DOUBLE SIMULTANEOUS FAULTS

Simultaneous faults occurring at two separate and distinct points on a three-phase system may involve any combination of six conductors and any combination of the short-circuit type faults listed on page 8. Since the current and voltage at each fault depend upon the current and voltage at the other fault, simultaneous faults may not be treated independently. Nevertheless, each fault may be considered separately in determining the relations existing between the symmetrical components of fault current and voltage at each fault point.

In Figure 4 let X and Y represent the two short-circuited points with a and A, b and B, and c and C being conductors on the same phase. V_a, V_b, V_c and V_A, V_B, V_C indicate line-to-ground voltages on phases a, b, c and

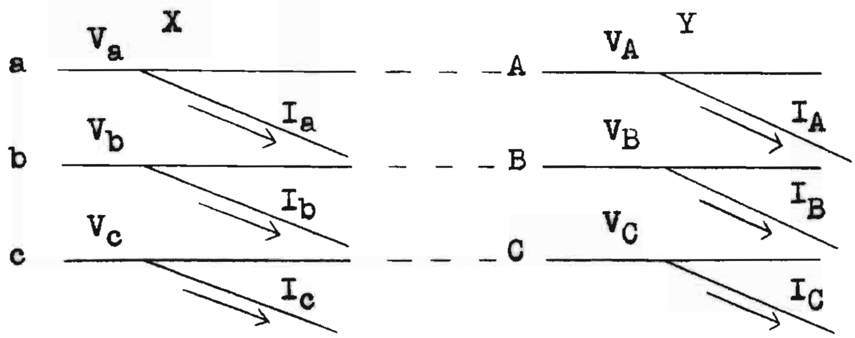


Fig. 4. Simultaneous faults at X and Y on a symmetrical three-phase system.

A, B, C, respectively, and I_a, I_b, I_c and I_A, I_B, I_C, the corresponding fault currents. The six symmetrical components of current and voltage at C are I_{x1}, I_{x2}, I_{xo},

V_{x1}, V_{x2}, and V_{xo} and those at D are I_{y1}, I_{y2}, I_{yo}, V_{y1}, V_{y2}, and V_{yo}. Twelve independent equations are necessary for the determination of the twelve unknown components. Two equations may be obtained from a consideration of the positive-sequence network impedances, and the relations between the positive-sequence components of fault current and voltage at each fault location. Similiarly, four more equations may be obtained, two from each of the negative- and zero-sequence networks. These six equations may be formed only because the system is assumed to be balanced, with the result that the three sequence-networks are independent of each other.

Three more equations, independent of the system impedances, may be developed for each fault point. These equations, which relate the components of voltage and current for the different sequences, depend only on the conditions existing at the fault. Such equations and the method for their determination were presented in the preceding discussion of single faults. These results were tabulated on page 18.

When the zero-sequence system may be represented by a one-line impedance diagram,[10] it is always possible to reduce it to an equivalent Y connecting the two fault points and the zero-potential bus. The simplest equiv-

(10) Clark, Edith, Simultaneous Faults on Three-Phase Systems, Jour. A.I.E.E., Vol. 50, Sept.1931, p936.

alent forms will in all cases be determined by the number of terminals whose identity must be retained. In general, the reduction may be effected by simple delta-wye or wye-delta transformations, but for complicated networks it may be necessary to use an a-c[11] or d-c calculating table.

Figure 5 shows the equivalent wye replacing the zero-sequence network between the two faults and the zero-potential bus. The branch impedances are X_o, Y_o, and S_o, and they may be either inductive or capacitive. Before the simultaneous faults occur, there are no currents nor voltages in the zero-sequence network. The effect of the simultaneous faults is to change the voltages at X and Y from zero to V_{xo} and from zero to V_{yo}, respectively, and the fault currents from zero to I_{xo} and zero to I_{yo}. Taking the positive direction of current flow as into the faults, the zero-sequence rise in voltage from the zero-potential bus to the points X and Y may be expressed as follows:

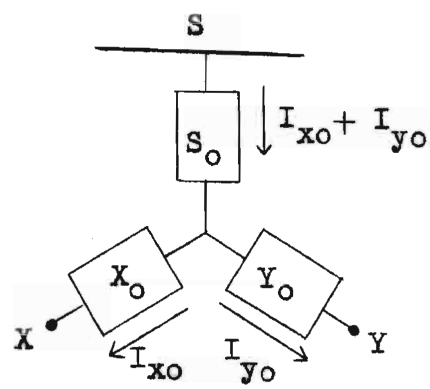


Fig. 5. Equivalent wye to replace zero sequence network between fault points X and Y and zero-potential bus S.

(11) Hazen, H. I., Schurig, O. R., Gardner, M. F., The M. I. T. Network Analyzer, A.I.E.E. Trans., 1930.

$$V_{xo} = -(I_{xo} + I_{yo})S_o - I_{xo}X_o = -I_{xo}(X_o + S_o) - I_{yo}S_o \qquad (17)$$

$$V_{yo} = -(I_{xo} + I_{yo})S_o - I_{yo}Y_o = -I_{xo}S_o - I_{yo}(Y_o + S_o) \qquad (18)$$

The negative-sequence voltages at the fault points may be obtained in a similiar manner from a consideration of Figure 6.

$$V_{x2} = - I_{x2}(X_2 \quad S_2) - I_{y2} S_2 \qquad (19)$$

$$V_{y2} = - I_{x2} S_2 - I_{y2}(Y_2 \quad S_2) \qquad (20)$$

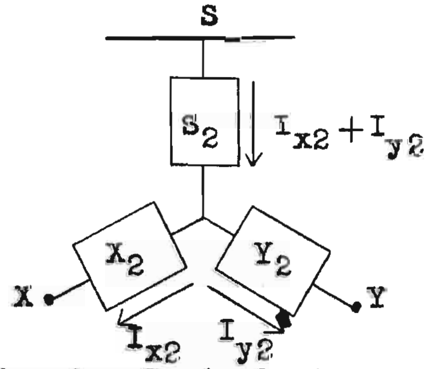


Fig. 6. Equivalent wye to replace negative-sequence network between fault points X and Y and zero-potential bus.

The positive-sequence system will differ from the negative- and zero-sequence systems since it will contain positive-sequence generated voltages. Let it be assumed that operating conditions existing before the occurrence of the faults are known and that the positive-sequence voltages at X and Y are V_f and V_F, respectively. This condition is represented by the equivalent positive-sequence wye(12) shown in Figure 7. When the fault occurs, the voltages at X and Y will become V_{x1} and V_{y1}, respectively. The positive-sequence fault currents, which were zero before the faults, will become some value I_{x1} at C and I_{y1} at D.

(12) Clarke, Edith, Circuit Analysis of A-C Power Systems, Vol. I, pp.202-204.

Therefore, the voltage rise from the zero-potential bus to the fault point X is

$$V_{x1} = V_f - I_{x1}(X_1 + S_1) - I_{y1} S_1 \tag{21}$$

Likewise, the voltage V_{y1} at fault point Y is

$$V_{y1} = V_F - I_{x1} S_1 - I_{y1}(Y_1 + S_1) \tag{22}$$

Equations (17), (18), (19), (20), (21), and (22) together with six equations from Table I afford the necessary relations to determine the twelve unknown components.

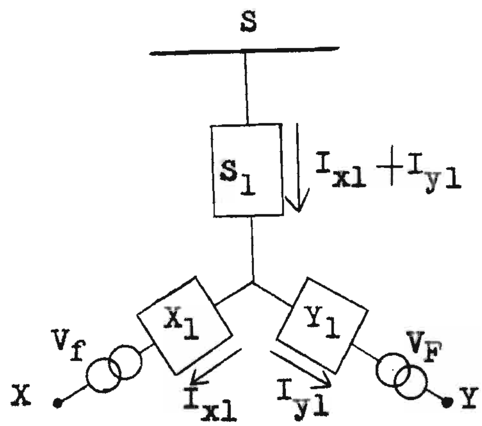


Fig. 7. Equivalent wye to replace the positive sequence network between fault points X and Y and zero-potential bus S.

THREE SIMULTANEOUS FAULTS

For the case of two simultaneously occurring faults, it is possible to reduce the positive-, negative-, and zero-sequence systems by network reduction methods to equivalent wyes or deltas, connecting the two fault points and the zero-potential bus. Expressions for the positive-, negative-, and zero-sequence potentials may then readily be written in terms of the positive-, negative-, and zero-sequence branch impedances, respectively. In reducing a network on which there are three simultaneously occurring faults, it is necessary to retain the identity of four points, namely, the three fault points and the zero-potential bus. The simplest equivalent circuit which may represent a sequence network in this case is a four terminal mesh(13) in which every terminal must connect every other terminal through a branch impedance. The six branch impedances so required may be obtained by means of an alternating-current calculating board, after which equations for sequence currents may be written in terms of the various network voltages and impedances.

Expressions for sequence voltages and currents are often simplified if a set of constants known as drop constants(14) are utilized. These constants will now be

---

(13) Lyon, W.V., Applications of the Method of Symmetrical Components, p. 48.
(14) Wagner and Evans, Symmetrical Components, pp241-243.

defined. Consider the four terminal network of Figure 7, connections being made as shown by means of an a-c calculating board. With a voltage impressed between X and the zero-potential bus S, current will flow through the network with resulting voltages at X, Y, and Z. The drop constants are:

$$D_{xx} = \frac{E_x}{I_x} \qquad D_{xy} = \frac{E_y}{I_x} \qquad D_{xz} = \frac{E_z}{I_z}$$

D_{xx} is merely the impedance of the network between the point X and the zero-potential bus, or merely the driving point impedance of the network at point X. D_{xy} is the mutual impedance existing between the circuit containing fault point X and that containing fault point Y; D_{xz} is the mutual impedance between the circuits of X and Z. By inserting voltages alternately at Y and Z and taking voltage and current readings as before, the other

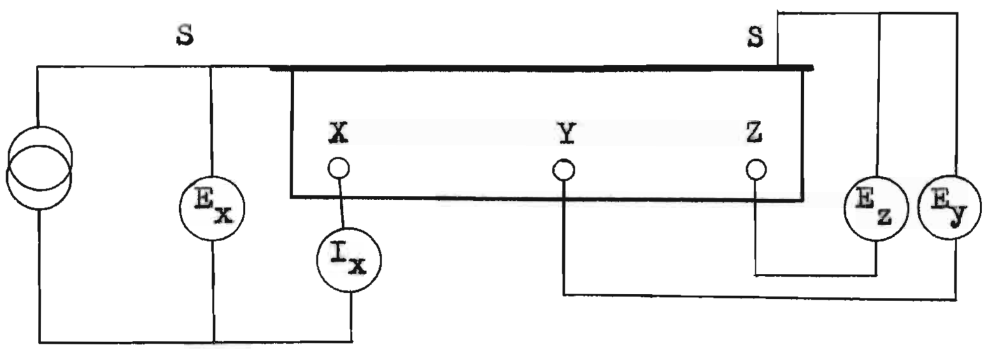


Fig. 8. Determination of drop constants by means of an a-c calculating board.

drop constants, D_{yy}, D_{yz}, and D_{zz} may be determined. Thus, for a four terminal network with a source of emf inserted between the zero-potential bus S and the network, the voltages existing at X, Y, and Z may be written as follows:

$$E_x = E - D_{xx}I_x - D_{xy}I_y - D_{xz}I_z \tag{23}$$

$$E_y = E - D_{xy}I_x - D_{yy}I_y - D_{yz}I_z \tag{24}$$

$$E_z = E - D_{xz}I_x - D_{yz}I_y - D_{zz}I_z \tag{25}$$

Let the three fault points on a symmetrical three-phase system be X, Y, and Z with the first, second, and third phases at points X, Y, and Z being a, b, and c, A, B, and C, and A', B', and C', respectively. Let V_a, V_b, V_c and V_A, V_B, V_C and V_A', V_B', V_C' be the voltages to ground on conductors a, b, and c at X, on A, B, and C at Y, and on A', B', and C' at Z and I_a, I_b, I_c and I_A, I_B, I_C and I_A', I_B', I_C' be the corresponding fault currents, positive direction for all currents and component currents being taken as into the fault.

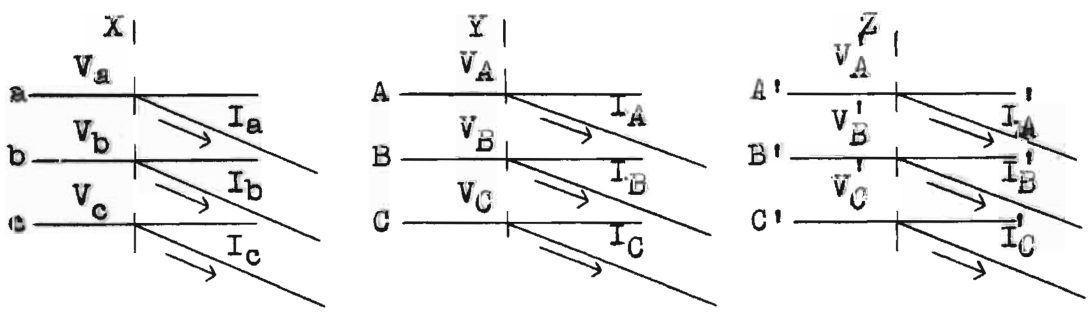


Fig. 9. Simultaneous faults at X, Y, and Z on a symmetrical three-phase system.

Let the symmetrical components of I_a and V_a be I_{x1}, I_{x2}, I_{xo} and V_{x1}, V_{x2}, V_{xo}, respectively; let those of

I_A and V_A be I_{y1}, I_{y2}, I_{yo} and V_{y1}, V_{y2}, V_{yo}, respectively; and let those of I_A' and V_A' be I_{z1}, I_{z2}, I_{zo} and V_{z1}, V_{z2}, V_{zo}, respectively.

Since six unknown components exist at each fault point, and since three faults are involved, eighteen independent equations will be required for a solution of the eighteen unknown quantities.

It has previously been determined that at each fault point three equations, independent of sequence impedances and depending only on conditions at the fault, may be developed to relate the three sequence currents and voltages. Equations of this type were discussed on pages 15, 16, and 17 with summarized results presented in Table I. Thus, when fault conditions are known, Table I may be used to furnish nine of the requisite equations to be used in the determination of the eighteen unknowns.

Three additional equations, expressing the sequence voltage at each fault point in terms of the sequence impedances and currents, may be determined from each of the three sequence networks thus supplying the other nine necessary equations. This will now be done for the negative- and zero-sequence networks.

Let it be assumed that by means of an a-c calculating board the drop constants associated with the negative-sequence network have been determined as D_{xx2}, D_{yy2},

D_{zz2}, D_{xy2}, D_{yz2}. Equations expressing the negative-sequence voltage at each of the three fault points in terms of the negative-sequence drop constants and negative-sequence currents may now be written as follows:

$$V_{x2} = -D_{xx2}I_{x2} - D_{xy2}I_{y2} - D_{xz2}I_{z2} \tag{26}$$

$$V_{y2} = -D_{xy2}I_{x2} - D_{yy2}I_{y2} - D_{yz2}I_{z2} \tag{27}$$

$$V_{z2} = - D_{xz2}I_{x2} - D_{yz2}I_{y2} - D_{zz2}I_{z2} \tag{28}$$

By an analogous procedure the zero-sequence voltage equations may also be written.

$$V_{xo} = -D_{xxo}I_{xo} - D_{xyo}I_{yo} - D_{xzo}I_{zo} \tag{29}$$

$$V_{yo} = -D_{xyo}I_{xo} - D_{yyo}I_{yo} - D_{yzo}I_{zo} \tag{30}$$

$$V_{zo} = -D_{xzo}I_{xo} - D_{yzo}I_{yo} - D_{zzo}I_{zo} \tag{31}$$

By utilizing nine equations from Table I together with equations (26), (27), (28), (29), (30), and (31) all negative- and zero-sequence components may be eliminated producing expressions for the positive-sequence fault voltages in terms of the three positive-sequence currents. The three resulting equations may be put in the following form:

$$V_{x1} = k\, I_{x1} + l\, I_{y1} + m\, I_{z1} \tag{32}$$

$$V_{y1} = n\, I_{x1} + o\, I_{y1} + p\, I_{z1} \tag{33}$$

$$V_{z1} = q\, I_{x1} + r\, I_{y1} + s\, I_{z1} \tag{34}$$

The constants k, l, m, n, o, p, q, r, and s are entirely independent of the positive-sequence network, and depend only on the impedances associated with the negative- and zero-sequence networks.

The method of eliminating the negative- and zero-sequence components and obtaining above mentioned constants will now be illustrated by examples.

Line-to-Ground Fault at points X, Y, and Z on phases a, A, and A', respectively. The following nine equations may be obtained from Table I. At fault point X

$$I_{xo} = I_{x1} \tag{35}$$

$$I_{x2} = I_{x1} \tag{36}$$

$$V_{x1} = -V_{xo} - V_{x2} \tag{37}$$

For the fault at Y

$$I_{yo} = I_{y1} \tag{38}$$

$$I_{y2} = I_{y1} \tag{39}$$

$$V_{y1} = -V_{yo} - V_{y2} \tag{40}$$

For the fault at Z

$$I_{zo} = I_{z1} \tag{41}$$

$$I_{z2} = I_{z1} \tag{42}$$

$$V_{z1} = -V_{zo} - V_{z2} \tag{43}$$

Equations (26)-(31) together with (35)-(43) are the fifteen equations to be solved. Adding equations (26) and (29) and replacing all negative- and zero-sequence components by corresponding positive-sequence values as given in equations (35)-(43),

$$V_{x1} = I_{x1}(D_{xx2} + D_{xxo}) + I_{y1}(D_{xy2} + D_{xyo}) + I_{z1}(D_{xz2} + D_{xzo}) \qquad (44)$$

Adding equations (27) and (30) and substituting for all negative- and zero-sequence components,

$$V_{y1} = (D_{xy2} + D_{xyo})I_{x1} + (D_{yy2} + D_{yyo})I_{y1} + (D_{yz2} + D_{yzo})I_{z1} \qquad (45)$$

Adding equations (28) and (31) and substituting for all negative- and zero-sequence components,

$$V_{z1} = (D_{xz2} + D_{xzo})I_{x1} + (D_{yz2} + D_{yzo})I_{y1} + (D_{zz2} + D_{zzo}) \qquad (46)$$

From equations (44), (45), and (46)

$$k = D_{xx2} + D_{xxo} \qquad (47)$$

$$l = D_{xy2} + D_{xyo} \qquad (48)$$

$$m = D_{xz2} + D_{xzo} \qquad (49)$$

$$n = D_{xy2} + D_{xyo} \qquad (50)$$

$$o = D_{yy2} + D_{yyo} \qquad (51)$$

$$p = D_{yz2} + D_{yzo} \qquad (52)$$

$$q = D_{xz2} + D_{xzo} \qquad (53)$$

$$r = D_{yz2} + D_{yzo} \qquad (54)$$

$$s = D_{zz2} + D_{zzo} \qquad (55)$$

These values for k, l, m, n, o, p, q, r, and s are the same as those shown in Table II for a single line-to-ground fault on phases a, A, and A'.

Line-to-Line Fault at points X, Y, and Z involving phases b and c, B and C, and B' and C'. Table I shows for a line-to-line fault at X involving phases b and c that

$$I_{xo} = 0 \tag{56}$$

$$I_{x2} = -I_{x1} \tag{57}$$

$$V_{x2} = V_{x1} \tag{58}$$

At fault point Y

$$I_{yo} = 0 \tag{59}$$

$$I_{y2} = -I_{y1} \tag{60}$$

$$V_{y2} = V_{y1} \tag{61}$$

At fault point Z

$$I_{zo} = 0 \tag{62}$$

$$I_{z2} = -I_{z1} \tag{63}$$

$$V_{z2} = V_{z1} \tag{64}$$

Substituting V_{x1} for V_{x2}, V_{y1} for V_{y2}, V_{z1} for V_{z2}, $-I_{x1}$ for I_{x2}, $-I_{y1}$ for I_{y2}, and $-I_{z1}$ for I_{z2} in equations (26), (27), and (28)

$$V_{x1} = D_{xx2}\, I_{x1} + D_{xy2}\, I_{y1} + D_{xz2}\, I_{z1} \tag{65}$$

$$V_{y1} = D_{xy2}\, I_{x1} + D_{yy2}\, I_{y1} + D_{yz2}\, I_{z1} \tag{66}$$

$$V_{z1} = D_{xz2}\, I_{x1} + D_{yz2}\, I_{y1} + D_{zz2}\, I_{z1} \tag{67}$$

k, l, m, n, o, p, q, r, and s are expressed by eqs. (65)-(67).

Three-Phase Fault involving phases a, b, and c at X, A, B, and C at Y, and A', B', and C' at Z. From Table I

$$I_{xo} = I_{yo} = I_{zo} \tag{68}$$

$$V_{x1} = V_{y1} = V_{z1} \tag{69}$$

$$V_{x2} = V_{y2} = V_{z2} \tag{70}$$

Since $V_{x1} = V_{y1} = V_{z1}$ and since I_{x1}, I_{y1}, and I_{z1} are not equal to zero, it is easily seen from equations (32), (33), and (34) that k, l, m, n, o, p, q, r, and s are all equal to zero. The results are unchanged if one, two, or all three of the three-phase faults are to ground.

Single Line-to-Ground Faults at X and Y on phases a and A, respectively, with a Line-to-Line Fault at Z on phases B' and C'. At fault point X

$$I_{xo} = I_{x1} \tag{71}$$

$$I_{x2} = I_{x1} \tag{72}$$

$$V_{x1} = -V_{xo} - V_{x2} \tag{73}$$

At fault point Y

$$I_{yo} = I_{y1} \tag{74}$$

$$I_{y2} = I_{y1} \tag{75}$$

$$V_{y1} = -V_{yo} - V_{y2} \tag{76}$$

At fault point Z

$$I_{zo} = 0 \tag{77}$$

$$I_{z2} = -I_{z1} \tag{78}$$

$$V_{z2} = V_{z1} \tag{79}$$

Adding equations (26) and (29) and substituting for all negative- and zero-sequence components using eqs.(71)-(79)

$$V_{x1} = (D_{xx2} + D_{xxo})I_{x1} + (D_{xy2} + D_{xyo})I_{y1} + D_{xz2}I_{z2} \tag{80}$$

Adding eqs. (27) and (30) and substituting for the negative- and zero-sequence components

$$V_{y1} = (D_{xy2} + D_{xyo})I_{x1} + (D_{yy2} + D_{yyo})I_{y1} + D_{yz2}I_{z1} \tag{81}$$

Adding eqs. (28) and (31) and substituting for the negative- and zero-sequence components

$$V_{z1} = (D_{xz2} + D_{xzo})I_{x1} + (D_{yz2} + D_{yzo})I_{y1} + D_{zz2}I_{z1} \tag{82}$$

Constants k, l, m, n, o, p, q, r, and s are expressed in equations (80), (81), and (82).

Single Line-to-Ground Fault at X on phase a with Line-to-Line Faults at Y and Z on phases B and C, and B' and C', respectively. From Table I at fault point X

$$I_{xo} = I_{x1} \tag{83}$$

$$I_{x2} = I_{x1} \tag{84}$$

$$V_{x1} = - V_{xo} - V_{x2} \tag{85}$$

At fault point Y

$$I_{yo} = 0 \tag{86}$$

$$I_{y2} = - I_{y1} \tag{87}$$

$$V_{y2} = V_{y1} \tag{88}$$

At fault point Z

$$I_{zo} = 0 \tag{89}$$

$$I_{z2} = - I_{z1} \tag{90}$$

$$V_{z2} = V_{z1} \tag{91}$$

Adding equations (26) and (29) and substituting for all negative- and zero-sequence components using eqs.(83)-(91)

$$V_{x1} = (D_{xx2} + D_{xxo})I_{x1} - D_{xy2}I_{y1} - D_{xz2}I_{z1} \tag{92}$$

Substituting equations (83)-(91) in equation (27)

$$V_{y1} = -D_{xy2}I_{x1} + D_{yy2}I_{y1} + D_{yz2}I_{z1} \tag{93}$$

Substituting equations (83)-(91) in equation 28

$$V_{z1} = -D_{xz2}I_{x1} + D_{yz2}I_{y1} + D_{zz2}I_{z1} \tag{94}$$

All nine constants are expressed in equations (92)-(94).

If the simplifying assumption that all generated emf's are equal and in phase is made, the positive-sequence network may be treated in much the same manner as the negative- and zero-sequence networks. Neglecting, for the moment, all generated emf's, the positive-sequence mutual and self impedance drop constants may be obtained by means of an a-c calculating board, circuit connections being made as shown in Figure 8. Then, if operating conditions existing before the occurrence of the faults are known, the positive-sequence voltages at the faults may be written in terms of the pre-fault voltages and the positive-sequence impedances. If the pre-fault voltages at X, Y, and Z are V_f, V_F, and V_F', respectively, the positive-sequence fault voltages are

$$V_{x1} = V_f - D_{xx1}I_{x1} - D_{xy1}I_{y1} - D_{xz1}I_{z1} \tag{95}$$

$$V_{y1} = V_F - D_{xy1}I_{x1} - D_{yy1}I_{y1} - D_{yz1}I_{z1} \tag{96}$$

$$V_{z1} = V_F' - D_{xz1}I_{x1} - D_{yz1}I_{y1} - D_{zz1}I_{z1} \tag{97}$$

Equations (32), (33), (34), (95), (96), and (97) express positive-sequence voltages in terms of positive-sequence currents only. The simultaneous solution of these six equations results in a complete determination of all positive-sequence components as shown below. Subtracting equation (32) from equation (95)

$$V_f = (D_{xx1} + k)I_{x1} + (D_{xy1} + l)I_{y1} + (D_{xz1} + m)I_{z1} \qquad (98)$$

Subtracting equation (33) from equation (96)

$$V_F = (D_{xy1} + n)I_{x1} + (D_{yy1} + o)I_{y1} + (D_{yz1} + p)I_{z1} \qquad (99)$$

Subtracting equation (34) from equation (97)

$$V_F' = (D_{xz1} + q)I_{x1} + (D_{yz1} + r)I_{y1} + (D_{zz1} + s)I_{z1} \qquad (100)$$

Equations (98), (99), and (100) may now be solved using determinates for the three currents, I_{x1}, I_{y1}, and I_{z1}.

$$I_{x1} = \frac{\begin{vmatrix} V_f & (D_{xy1} \quad l) & (D_{xz1} \quad m) \\ V_F & (D_{yy1} \quad o) & (D_{yz1} \quad p) \\ V_F' & (D_{yz1} \quad r) & (D_{zz1} \quad s) \end{vmatrix}}{\begin{vmatrix} (D_{xx1} \quad k) & (D_{xy1} \quad l) & (D_{xz1} \quad m) \\ (D_{xy1} \quad n) & (D_{yy1} \quad o) & (D_{yz1} \quad p) \\ (D_{xz1} \quad q) & (D_{yz1} \quad r) & (D_{zz1} \quad s) \end{vmatrix} = D}$$

$$I_{x1} = \frac{\left[(D_{yy1} + o)(D_{zz1} + s) - (D_{yz1} + p)(D_{yz1} + r)\right] V_f}{D} -$$

$$\frac{\left[(D_{xy1} + l)(D_{zz1} + s) - (D_{xz1} + m)(D_{yz1} + r)\right] V_F}{D} -$$

$$\frac{\left[(D_{xy1} + l)(D_{yz1} + p) - (D_{xz1} + m)(D_{yy1} + o)\right] V_F'}{D} \qquad (101)$$

where

$$D = (D_{xx1} + k) \quad (D_{yy1} + o)(D_{zz1} + s) - (D_{yz1} + p)(D_{yz1} + r) \;—$$

$$(D_{xy1}+n)\left[(D_{xy1}+l)(D_{zz1}+s) - (D_{xz1}+m)(D_{yz1}+r)\right] -$$
$$(D_{xz1}+q)\left[(D_{xy1}+l)(D_{yz1}+p) - (D_{xz1}+m)(D_{yy1}+o)\right]$$

Solving for I_{y1} in a similiar manner,

$$I_{y1} = \frac{-V_f\left[(D_{xy1}+n)(D_{zz1}+s) - (D_{yz1}+p)(D_{xz1}+q)\right]}{D} - \frac{V_F\left[(D_{xx1}+k)(D_{zz1}+s) - (D_{xz1}+m)(D_{xz1}+q)\right]}{D} - \frac{V_F'\left[(D_{xx1}+k)(D_{yz1}+p) - (D_{xz1}+m)(D_{xy1}+n)\right]}{D} \quad (102)$$

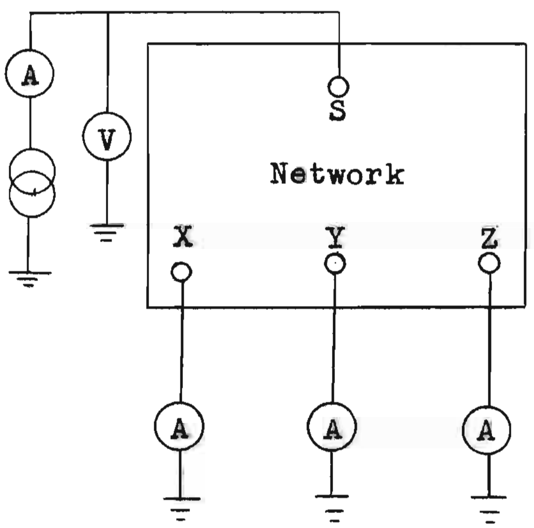
Solving for I_{z1} ,

$$I_{z1} = \frac{V_f\left[(D_{xy1}+n)(D_{yz1}+r) - (D_{yy1}+o)(D_{xz1}+q)\right]}{D} - \frac{-V_F\left[(D_{xx1}+k)(D_{yz1}+r) - (D_{xy1}+l)(D_{xz1}+q)\right]}{D} - \frac{V_F'\left[(D_{xx1}+k)(D_{yy1}+o) - (D_{xy1}+l)(D_{xy1}+n)\right]}{D} \quad (103)$$

To evaluate V_{x1}, V_{y1}, and V_{z1} it is only necessary to substitute the results of equations (101), (102), and (103) into equations (32), (33), and (34).

Having evaluated the six positive-sequence components, all negative- and zero-sequence quantities may be found. These results substituted in equations (4), (5), and (6) completely determine the three fault currents and three fault voltages.

Sometimes it is not desirable to assume all generated emf's to be equal and in phase. Since all generated voltages are positive-sequence voltages, the negative- and zero-sequence voltages at the faults, nevertheless, may still be expressed by equations (26)-(31). It is only the positive-sequence equations, which relate positive-sequence voltages and currents, that must assume a different form. Before these equations can be developed, it is necessary to consider certain network simplifications which may be effected by means of an a-c calculating board. Consider the four terminal network of Figure 10. As previously stated, the simplest equiv-



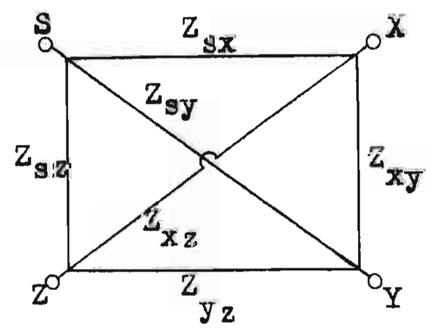


Fig. 10. Network simplification using a-c calculating board.

Fig. 11. Equivalent circuit to replace four-terminal network.

alent circuit which may replace any four terminal network is another four terminal network in which every terminal is connected to every other through a branch impedance. Figure 11 shows the equivalent circuit which is to replace the four terminal network of Figure 10. For the measurements indicated in Figure 10, the following impedances may be obtained.

$$Z_{sx} = V/I_x \qquad Z_{sy} = V/I_y \qquad Z_{sz} = V/I_z$$

By applying an emf at other terminals and repeating the process, the other three branch impedances may be obtained.

With two sources of emf(15) the positive-sequence drop constants are no longer used. Rather equations for the positive-sequence currents are written in terms of the positive-sequence fault voltages and the positive-sequence branch impedances Z_{sx1}, Z_{sy1}, Z_{sz1}, Z_{xy1}, $Z_{xz,}$, and Z_{yz1}. If afain the positive direction of current is assumed to be out at the network terminals and into the fault, the positive-sequence currents are

$$I_{x1} = \frac{V_{s1} - V_{x1}}{Z_{sx1}} + \frac{V_{y1} - V_{x1}}{Z_{xy1}} + \frac{V_{z1} - V_{x1}}{Z_{xz1}} \qquad (104)$$

$$I_{y1} = \frac{V_{s1} - V_{y1}}{Z_{sy1}} + \frac{V_{x1} - V_{y1}}{Z_{xy1}} + \frac{V_{z1} - V_{y1}}{Z_{yz1}} \qquad (105)$$

$$I_{z1} = \frac{V_{s1} - V_{z1}}{Z_{sz1}} + \frac{V_{x1} - V_{z1}}{Z_{xz1}} + \frac{V_{y1} - V_{z1}}{Z_{yz1}} \qquad (106)$$

For this case of two emf's the above equations replace equations (95), (96), and (97) in the solution of the positive-sequence components. Thus, simultaneous solution of equations (104), (105), and (106) together with equations (32), (33), and (34) provide the six equations necessary to determine the six positive-sequence components.

(15) Wagner and Evans, Symmetrical Components, p. 245.

TABLE II

Values of k, l, m, n, o, p, q, r, and s for Simultaneous Faults at Three Points on a Symmetrical Three-Phase System. Phase a, A, and A' are reference phases.

Case A. Single Line-to-Ground Faults at Three Points.

(a) Phases a, A, and A'

$k = (D_{xx2} + D_{xxo})$

$l = (D_{xy2} + D_{xyo})$

$m = (D_{xz2} + D_{xzo})$

$n = (D_{xy2} + D_{xyo})$

$o = (D_{yy2} + D_{yyo})$

$p = (D_{yz2} + D_{yzo})$

$q = (D_{xz2} + D_{xzo})$

$r = (D_{yz2} + I_{yzo})$

$s = (D_{zz2} + D_{zzo})$

(b) Phases a, A, and B'

$k = (D_{xx2} + D_{xxo})$

$l = (D_{xy2} + D_{xyo})$

$m = (aD_{xz2} + a^2D_{xzo})$

$n = (D_{xy2} + D_{xyo})$

$o = (D_{yy2} + D_{yyo})$

$p = (aD_{yz2} + a^2D_{yzo})$

$q = (a^2D_{xz2} + aD_{xzo})$

$r = (a^2D_{yz2} + aD_{yzo})$

$s = (D_{zz2} + D_{zzo})$

(c) Phases a, A, and C'

$k = (D_{xx2} + D_{xxo})$

$l = (D_{xy2} + D_{xyo})$

$m = (a^2D_{xz2} + aD_{xzo})$

$n = (D_{xy2} + D_{xyo})$

$o = (D_{yy2} + D_{yyo})$

$p = (a^2D_{yz2} + aD_{yzo})$

$q = (aD_{xz2} + a^2D_{xzo})$

$r = (aD_{yz2} + a^2D_{yzo})$

$s = (D_{zz2} + D_{zzo})$

(d) Phases a, B, and A'

$k = (D_{xx2} + D_{xxo})$

$l = (aD_{xy2} + a^2D_{xyo})$

$m = (D_{xz2} + D_{xzo})$

$n = (a^2D_{xy2} + aD_{xyo})$

$o = (D_{yy2} + D_{yyo})$

$p = (a^2D_{yz2} + aD_{yzo})$

$q = (D_{xz2} + D_{xzo})$

$r = (aD_{yz2} + a^2D_{yzo})$

$s = (D_{zz2} + D_{zzo})$

TABLE II cont'd

Case B. Line-to-Line Faults at Three Points.

(a) Phases b and c, B and C, B' and C'

$k = D_{xx2}$

$l = D_{xy2}$

$m = D_{xz2}$

$n = D_{xy2}$

$o = D_{yy2}$

$p = D_{yz2}$

$q = D_{xz2}$

$r = D_{yz2}$

$s = D_{zz2}$

(b) Phases a and c, B and C, and B' and C'.

$k = D_{xx2}$

$l = a^2D_{xy2}$

$m = a^2D_{xz2}$

$n = aD_{xy2}$

$o = D_{yy2}$

$p = D_{yz2}$

$q = aD_{xz2}$

$r = D_{yz2}$

$s = D_{zz2}$

(c) Phases a and b, B and C, B' and C'

$k = D_{xx2}$

$l = aD_{xy2}$

$m = aD_{xz2}$

$n = a^2D_{xy2}$

$o = D_{yy2}$

$p = D_{yz2}$

$q = a^2D_{xz2}$

$r = D_{yz2}$

$s = D_{zz2}$

(d) Phases b and c, A and C, and B' and C'

$k = D_{xx2}$

$l = aD_{xy2}$

$m = D_{xz2}$

$n = a^2D_{xy2}$

$o = D_{yy2}$

$p = a^2D_{yz2}$

$q = D_{xz2}$

$r = aD_{yz2}$

$s = D_{zz2}$

Case C. Three-Phase Faults at Three Points.

$$k = l = m = n = o = p = q = r = s = 0$$

TABLE II cont'd

Case D. Line-to-Ground Fault at two points with a Line-to-Line Fault at one point.

(a) Phases a, A, and A'andC'

$k = (D_{xx2} + D_{xxo})$

$l = (D_{xy2} + D_{xyo})$

$m = aD_{xz2}$

$n = (D_{xy2} + D_{xyo})$

$o = (D_{yy2} + D_{yyo})$

$p = aD_{yz2}$

$q = -a^2 D_{xz2}$

$r = -a^2 D_{yz2}$

$s = D_{zz2}$

(b) Phases a, A, and B'& C'

$k = (D_{xx2} + D_{xxo})$

$l = (D_{xy2} + D_{xyo})$

$m = -D_{xz2}$

$n = (D_{xy2} + D_{xyo})$

$o = (D_{yy2} + D_{yyo})$

$p = -D_{yz2}$

$q = (D_{xz2} + D_{xzo})$

$r = (D_{yz2} + D_{yzo})$

$s = -D_{zz2}$

(c) Phases a, B and C, A'

$k = (D_{xx2} + D_{xxo})$

$l = -D_{xy2}$

$m = (D_{xz2} + D_{xzo})$

$n = -D_{xy2}$

$o = D_{yy2}$

$p = -D_{yz2}$

$q = (D_{xz2} + D_{xzo})$

$r = -D_{yz2}$

$s = (D_{zz2} + D_{zzo})$

(d) Phases band c, A, A'

$k = D_{xx2}$

$l = -D_{xy2}$

$m = -D_{xz2}$

$n = -D_{xy2}$

$o = (D_{yy2} + D_{yyo})$

$p = (D_{yz2} + D_{yzo})$

$q = -D_{xz2}$

$r = (D_{yz2} + D_{yzo})$

$s = (D_{zz2} + D_{zzo})$

TABLE II cont'd

Case E. Line-to-Line Fault at two points and a Line-to-Ground Fault at one point.

(a) Phases a, B & C, B' & C'	(b) Phases b & c, A, B' & C'
$k = (D_{xx2} + D_{xxo})$	$k = D_{xx2}$
$l = -D_{xy2}$	$l = -D_{xy2}$
$m = -D_{xz2}$	$m = D_{xz2}$
$n = -D_{xy2}$	$n = -D_{xy2}$
$o = D_{yy2}$	$o = (D_{yy2} + D_{yyo})$
$p = D_{yz2}$	$p = D_{yz2}$
$q = -D_{xz2}$	$q = D_{xz2}$
$r = D_{yz2}$	$r = D_{yz2}$
$s = D_{zz2}$	$s = D_{zz2}$

SUMMARY

The method for the determination of fault currents and voltages on symmetrical three-phase systems subjected to one, two or three simultaneous faults was presented. The fault analysis was effected by the Method of Symmetrical Components.

Three types of short-circuit faults (line-to-ground, line-to-line, and three-phase) and combinations thereof were considered. Double line-to-ground faults were not considered since, for faults of this type, the solution is excessively laborious and the final results so extensive as to make them relatively useless.

BIBLIOGRAPHY

Books:

a. Clarke, Edith. Circuit Analysis of A-C Power Systems. N. Y., Wiley, 1943. pp. 55-224. Vol I
Vol II

b. Lyon, W. V. Applications of The Method of Symmetrical Components. 1st ed. N. Y., McGraw-Hill, 1937. pp. 487-489.

c. Wagner, C. F., and Evans, R. D. Symmetrical Components. 1st ed. N. Y., Mc-Graw-Hill, 1933. pp. 238-258.

W

Periodicals:

a. Clarke, Edith. Simultaneous Faults on Three-Phase Systems. A.I.E.E. Trans., Vol. 50, pp. 919-941 (1931)

b. Fortescue, C. L. Method of Symmetrical Coordinates Applied to the Solution of Polyphase Networks, Trans. A.E.E.E., Vol. 37, pp. 1027-1140 (1918)

c. Hazen, H. I., Schurig, O. R., and Gardner, M. F. The M. I. T. Network Analyzer, A.I.E.E. Trans.,(1930)

d. Lamme, B. B. The Story of the Induction Motor. Jour. A.I.E.E., Vol. 40, pp 669-671 (1921)

e. Lewis, W. W. A New Short-Circuit Calculating Table. Gen. Elec. Rev., pp. 693-716 (1920)

Publications of Learned Societies:

a. Carson. Engineering Reports of the Joint Subcommittee on Development and Research. Nat. Elec. Light Assoc. and Bell Tele. Sys. Vol. I, Engineering Report 4, pp. 7-47.

Thesis

Foreigh Publications:

a. Ferraris. Rotazioni elettrodynamiche. Turin Acad., March 1888.

b. Stokvis, L. G., and Oldenbourg, R. Der Spannungsbfall des synchronen Drehstrom-generators bei unsymmetrischer Belastung. Munich. 1922.

c. Stokvis, L. G. Sur la creation des harmoniques 3 dans les alternateurs par suite des desequilibrages des phases. Comp. rend. Vol. 159, 1914. pp. 46-49.

VITA

Robert M. Montgomery was born in Sikeston, Missouri on November 16, 1923. Elementary and secondary schooling was received Sikeston, and a Bachelor of Science in Electrical Engineering was obtained in August 1944 from the Missouri School of Mines and Metallurgy, University of Missouri at Rolla, Missouri.